Fourier series - THEORY

Dirichlet conditions

It is said that the function f(x) meets Dirichlet conditions in the interval (a,b) if

i) uniformly limited, that is $|f(x)| \leq M$ **for each** $x \in (a,b)$**, where M is constant**

(If it is limited in the interval *(a, b)* or the final at all points of the interval)

ii) there are no more than a finite number of break points and they are all first-order, that is, at any ends point there are finite left and right limit

(If the interval is continuous or has a finite number of ends of the first kind, the so-called "jumping")

iii) there are no more than a finite number of real extremum

(If in a given interval monotone or has a finite number of extremum)

Dirichlet theorem

Let $f(x) \in X`, X` = \{f \in X \,/\, f \text{ have adequate one-sided derivatives on } [-\pi,\pi]\}$.

Fourier series of f (x) converges to values:

$$\frac{f(x-0)+f(x+0)}{2}$$

and in points $x = \pm\pi$ converges to :

$$\frac{f(\pi-0)+f(-\pi+0)}{2}$$

For example, if we take points b and c as interruption points of the first kind, Fourier series has sum:

$$S = \frac{f(b-0)+f(b+0)}{2} \quad \text{or} \quad S = \frac{f(c-0)+f(c+0)}{2}$$

For example , if we have interval [*m,n*], at the ends of the interval, the sum of Fourier series is $S = \frac{f(m)+f(n)}{2}$

How to developed function in Fourier series?

In the lectures , professors usually explain this over the function f (x) given in the interval $[-\pi,\pi]$, which is

periodically extended with period 2π .

Fourier series have form:

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

a_0,a_n,b_n are Fourier coefficients, which we just look over the formula

<u>1) If we are given the interval</u> $[-\pi,\pi]$

i) If the function is neither even nor odd, then:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

ii) If on the interval $[-\pi,\pi]$ function is odd, then:

$$b_n=\frac{2}{\pi}\int_{0}^{\pi}f(x)\sin nxdx$$ where are $a_0=0\wedge a_n=0$

$$f(x)=\sum_{n=1}^{\infty}b_n\sin nx$$

iii) If on the interval $[-\pi,\pi]$ function is even, then:

$$a_0=\frac{2}{\pi}\int_{0}^{\pi}f(x)dx$$

$$a_n=\frac{2}{\pi}\int_{0}^{\pi}f(x)\cos nxdx$$ while $b_n=0$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos nx$$

2) If we are given the interval $[-l, l]$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l})$$

3) If the function is given in an arbitrary interval [*a, b*]

$$a_0 = \frac{2}{b-a}\int_{a}^{b} f(x)dx$$

$$a_n = \frac{2}{b-a}\int_{a}^{b} f(x)\cos\frac{2n\pi x}{b-a}dx$$

$$b_n = \frac{2}{b-a}\int_{a}^{b} f(x)\sin\frac{2n\pi x}{b-a}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{2n\pi x}{b-a} + b_n \sin\frac{2n\pi x}{b-a})$$

The task will often fall some of the following integral, so that would not be tortured:

$$\int_{-\pi}^{\pi} \sin nx dx = 0; \quad \int_{-\pi}^{\pi} \cos nx dx = 0; \quad \int_{-\pi}^{\pi} \cos nx \sin nx dx = 0; \quad \int_{-\pi}^{\pi} \cos nx \cos mx dx = 0$$

$$\int_{-\pi}^{\pi} \sin nx \sin mx dx = 0; \quad \int_{-\pi}^{\pi} \cos^2 nx dx = \pi; \quad \int_{-\pi}^{\pi} \sin^2 nx dx = \pi$$

In all of these integrals is, of course $m \neq n$